

# Viscosity Effects in Cocurrent Three-Phase Fluidization

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Many industrial processes are based on cocurrent three-phase fluidization operations. Extended reviews (Fan, 1989; Muroyama and Fan, 1985; De Lasa and Lee, 1986; Wild et al., 1982; Epstein, 1981) have shown that most of the experimental investigations in this field deal with low-viscosity liquids. Reported in this note are experimental data on phase holdup in a cocurrent three-phase fluidized bed for moderately viscous liquids. Several existing empirical correlations are assessed, and modifications of existing correlations are proposed that account for liquid viscosity effects. The holdup data are also analyzed using bubble wake models with emphasis on their limitations.

## Experimental Apparatus

Experiments were performed in a 76.2 mm diameter and 1.1 m high plexiglass column. Glass beads of a diameter equal to 2, 3.9 or 6 mm and density equal to 2,420, 2,850 and 2,603 kg/m<sup>3</sup>, respectively, were used. The gas phase consisted of air, and the viscosity of the liquid phase was varied from 10 to 120 mPa · s by using mineral oils (Shell Vitrea 32 and 320) and mixtures of Vitrea oils with kerosene. The liquid phase had a Newtonian behavior and its density varied from 850 to 870 kg/m<sup>3</sup>.

The operating variables were the particle diameter ( $d_p$ ), the superficial gas and liquid velocities ( $U_G$ ,  $U_L$ ), and the liquid viscosity ( $\mu_L$ ). The first three covered the following ranges:

$$2 < d_p(\text{mm}) < 6, \quad 0 < U_G(\text{mm/s}) < 50, \quad 3 < U_L(\text{mm/s}) < 43,$$

For the porosity measurements, the liquid viscosity was

varied in the range:

$$10 < \mu_L(\text{mPa} \cdot \text{s}) < 124 \quad (1)$$

But for the liquid and gas holdup measurements, the viscosity was restricted to the range:

$$35 < \mu_L(\text{mPa} \cdot \text{s}) < 75 \quad (2)$$

The gas-liquid mixture entered the column through a distributor consisting of an inverted cone filled with 4 mm diameter glass beads separated from the column by a 19 mm thick perforated plate with 89 square pitched holes of 2 mm diameter. Bed height and pressure drop measurements were used to calculate phase holdups knowing the total weight of particles (Muroyama and Fan, 1985). A complete description of the experimental apparatus and procedure is given by Grandjean (1989).

## Liquid Holdup in Liquid-Solid Bed ( $U_G = 0$ )

In absence of gas sparging, the liquid holdup has been correlated by the relation of Richardson and Zaki (1954):

$$\epsilon_L^N = U_L/V_{MT} \quad (3)$$

with

$$\log(V_{MT}) = \log(U_T) - d_p/D_C \quad (4)$$

The terminal velocity of the particles,  $U_T$ , has been measured and was found to be well predicted using the trial-and-error procedure described by Bird et al. (1960). The exponent  $N$  suggested by Richardson and Zaki for the range of our experi-

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mental data is given by:

$$N = (4.4 + 18d_p/D_C)Re_T^{-0.1},$$

$$1 < Re_T = \rho_L U_T d_p / \mu_L < 200, \quad (5)$$

This relation, however, leads to liquid holdup predictions about 10% higher than the experimental values. We propose the following modified equation obtained by nonlinear regression analysis:

$$N = (3.0 + 63.5 d_p/D_C)Re_T^{-0.24}, \quad 0.5 < Re_T < 42 \quad (6)$$

### Predictions of Porosity in Three-Phase Fluidized Bed

Several correlations available in the literature (reviewed by Muroyama and Fan, 1985) have been tested. The average relative deviations between the 400 data of this investigation for the porosity and the predicted values using these correlations are shown in Table 1.

Depending on the particle diameter, the relative deviations vary from 4 to 72%. The best correlation in the literature appears to be that of Saberian et al. (1984). However, the deviations for the 2 mm particle data are quite large. Moreover, as shown in Figure 1, all the effects except for the viscosity are well described by the correlation of Begovich and Watson (1978), which is expressed by:

$$\epsilon = 3.93\mu_L^{0.055}U_L^{0.271}U_G^{0.041}(\rho_S - \rho_L)^{-0.316}d_p^{-0.268}D_C^{-0.033} \quad (7)$$

This correlation which is the most widely accepted empirical correlation for porosity (Fan, 1989) is based on the data of nine different investigations, of which eight were conducted with the liquids of viscosity lower than 12 mPa · s. Moreover, for six of these investigations, liquids of viscosity close to 1 mPa · s were used. We note from Figure 1 that the predicted porosity is half of the experimental value when the liquid viscosity reaches 120 mPa · s.

To extend the applicability of Begovitch-Watson correlation to higher viscosity liquids, we propose the following modification:

$$\epsilon = (2.5 + 13.2\mu_L^{0.64})U_L^{0.271}U_G^{0.041} \cdot (\rho_S - \rho_L)^{-0.316}d_p^{-0.268}D_C^{-0.033} \quad (8)$$

The coefficient 2.5 in the viscosity term has been introduced to recover the viscosity effect predicted by relation 7 when  $\mu_L$  is

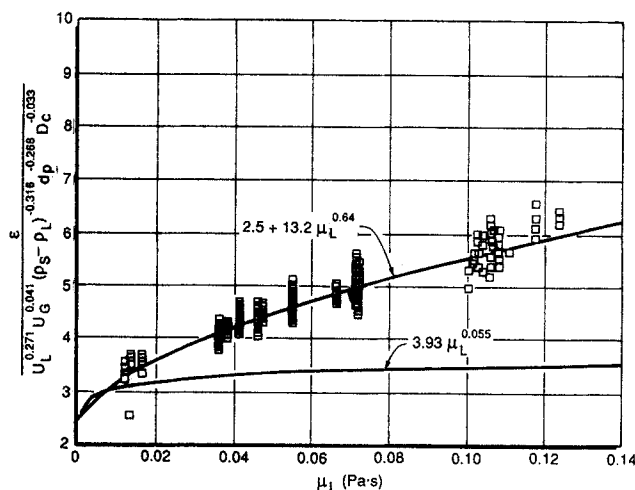


Figure 1. Effect of viscosity on the bed porosity.

close to 1 mPa · s. The computed values using Eq. 8 and our experimental data are compared in Figure 1. The correlation is quite good and the average deviations, reported in Table 1, are equal to 4% or less. This is a considerable improvement over the existing correlations.

### Predictions of Liquid Holdup

The average relative deviations between our liquid holdup data and calculated values from several existing correlations are reported in Table 2. The best predictions are obtained using the correlation of Kato et al. (1981). Their correlation contains 14 empirical coefficients and, as stressed by Saberian et al. (1984), the main drawback is its complicated form.

The simple correlation of Saberian et al. (1984) is based on a slip velocity  $U_{GL}$ , defined by:

$$U_{GL}/\epsilon_G = U_G/\epsilon_G - (U_G + U_L)/(\epsilon_L + \epsilon_G) \quad (9)$$

The following simple relationships for the liquid and gas holdups are then obtained:

$$\epsilon_L = \epsilon(U_L + U_{GL})/(U_G + U_L) \quad (10)$$

$$\epsilon_G = \epsilon(U_G - U_{GL})/(U_G + U_L) \quad (11)$$

Saberian et al. (1984) have correlated the slip velocity by:

$$U_{GL} = 0.017(\rho_L U_G^2)^{0.45} \quad (12)$$

Table 1. Porosity Predictions

Correlations	Avg. Relative Deviation, $d_p$ (mm)		
	2.0	3.9	6.0
Dakshinamurty et al. (1972)	0.72	0.67	0.52
Kim et al. (1975)	0.14	0.33	0.24
Soung (1978)	0.37	0.13	0.11
Saberian et al. (1984)	0.35	0.16	0.04
Begovich and Watson (1978), Eq. 7	0.26	0.25	0.28
This Work, Eq. 8	0.02	0.04	0.04

Table 2. Liquid Holdup Predictions

Correlations	Avg. Relative Deviations, $d_p$ (mm)		
	2.0	3.9	6.0
Kato et al. (1981)	0.14	0.12	0.07
Kim et al. (1975)	0.47	0.62	0.54
Razumov et al. (1973)	0.18	0.11	0.14
Saberian et al. (1984), Eqs. 8, 10, 12	0.34	0.38	0.36
This Work, Eqs. 8, 10, 13	0.21	0.27	0.24

A slightly modified form has been recently suggested by Wild et al. (1989). Relation 10, in which  $U_{GL}$  and  $\epsilon$  were computed using Eqs. 8 and 12, respectively, has been tested. The relative deviations are shown in Table 2 to vary from 34 to 36%. To explain the large deviations, we have computed  $U_{GL}$ , as defined by Eq. 9, using the experimental values for the holdups. As  $U_{GL}$  was found to slightly increase with the liquid viscosity, we propose the following modification of Eq. 12:

$$U_{GL} = 0.034\mu_L^{0.1}(\rho_L U_G^2)^{0.45} \quad (13)$$

Equation 13 reduces to Eq. 12 for a liquid viscosity equal to 1 mPa · s (viscosity value for most of the fluids investigated by Saberian et al., 1984). As shown in Table 2, the use of Eq. 13 instead of Eq. 12 improves the liquid holdup predictions considerably, but the correlation is not as satisfactory as those of Kato et al. (1981) and of Razumov et al. (1973). The viscosity effect predicted by Eq. 13 is, however, in contradiction with the findings of Patwari (cited by Wild et al., 1989), who reported no effect of the liquid viscosity on the slip velocity. Moreover, it should be stressed that Wild et al. (1989) found the slip velocity to be a function of the distributor geometry for small-scale columns. Hence, the applicability of Eq. 13 to various sizes and different distributor geometries needs to be verified.

### Prediction of Gas Holdup

The predictions using the literature correlations for the gas holdup show very large deviations. Table 3 shows large variations in the relative deviation for a given correlation depending on the particle diameter. This is partly due to changes in the bubble gas regime which are not accounted for in the correlations. This is critical for gas holdup. In our experiments, the 2 mm particle bed has shown an initial contraction when gas was injected in the liquid-solid bed. This is a characteristic of the coalesced bubble regime. On the contrary, an expansion of the bed was observed in the case of the 6 mm particles. This is a characteristic of the bubble disintegration regime. For the 3.9 mm particle bed, contraction or expansion was observed depending on the superficial liquid velocity and on the liquid viscosity. The 3.9 mm bed was operated in the transition region between the disintegrating and coalescing bubble regimes.

The gas holdup predictions from Eq. 11 are significantly improved when  $U_{GL}$  is calculated using Eq. 13 instead of Eq. 12. As shown in Table 3, the results are slightly better than those obtained with the correlation of Begovitch and Watson (1978), but much better than the predictions of the other correlations.

**Table 3. Gas Holdup Predictions**

Correlations	Avg. Relative Deviations, $d_p$ (mm)		
	2.0	3.9	6.0
Begovitch and Watson (1978)	0.58	0.22	0.20
Khang et al. (1983)	0.32	1.97	1.81
Ziganshin et al. (1970)	0.25	1.56	1.56
Saberian et al. (1984), Eqs. 8, 11, 12	0.79	1.75	1.38
This Work, Eqs. 8, 11, 13	0.36	0.22	0.26

### Bubble Wake Models

A generalized bubble wake model has been proposed by Bathia and Epstein (1974). The liquid holdup is then expressed by:

$$\epsilon_L = \left[ \frac{U_L - kU_G(1-x)}{U_T[1 - \epsilon_G(1+k)]} \right]^{1/N} \cdot \{1 - \epsilon_G[1 + k(1-x)]\} + (1-x)k\epsilon_G \quad (14)$$

where  $k$  is the ratio of the bubble wake volume to the bubble volume, and  $x$  is the ratio of the solids holdup in the wake to the solids holdup in the liquid-solid region,

$$k = \epsilon_W / \epsilon_G \quad (15)$$

and

$$x = \epsilon_{SW} / \epsilon_{SF} \quad (16)$$

As  $k$  or  $x$  has been observed to vary with operating conditions (Muroyama and Fan, 1985), additional assumptions are needed to simplify the analysis.

### No solid particles in the wake, $x = 0$

As assumed by many other investigators (Wild et al., 1982), we first set the solids holdup in the wake equal to zero. The values for  $k$  were then obtained from Eq. 14 using an iteration procedure. For the 2 mm particle bed,  $k$  was found to increase as the ratio  $(U_L/U_G)$  increased. This is qualitatively in agreement with the findings of Darton et Harrison (1975), who suggested the following relation:

$$k = 1.4(U_L/U_G)^{0.33} - 1 \quad (17)$$

However, the best fit of our data is the following:

$$k = 0.42(U_L/U_G)^{0.79} \quad (18)$$

For the 3.9 and 6 mm particle bed,  $k$  was found to vary between 0 and 1, but the data are scattered and no trends of variation with  $(U_L/U_G)$  or with any other variables have been observed. This suggests that the assumption of  $x$  equal to zero is not acceptable, at least for these particle diameters.

### Solid particles in the wake

The assumption of  $x$  equal to 1 was also examined, but most of the values of  $k$  were found to be negative. This has no physical meaning. Then, we used the results of Kitano et Fan (1988), who measured the solids holdup in the wake for beds of particle diameters equal to 0.46, 0.77 and 1 mm. They observed that the average solids holdup in the wake was close to 0.42. The parameter,  $x$ , is then given by:

$$x = 0.42 / \epsilon_{SF} \quad (19)$$

Inserting this expression in the original equations of the bubble wake model, the values for  $x$  and  $k$  were found to vary from 0.7 up to 1.4 and from  $-6$  up to 66, respectively. The range of variation for  $x$  is physically acceptable but not that for  $k$ . These

results show the difficulty of estimating the bubble wake parameters  $k$  and  $x$  from holdup data only. The pioneering work of Kitano and Fan (1988) on the direct measurements of bubble wake parameters is of considerable interest; and in a more recent publication, Kreischer et al. (1990) proposed a correlation for the wake solids holdup as a function of the bubble Reynolds number. The bubble Reynolds number, however, is not an operating parameter and the use of their correlation for design purpose is hence restricted.

The difficulty of estimating the bubble wake parameters  $k$  and  $x$  from the holdup data only is largely due to the small contributions of the wake to the overall solids and liquid holdups. Indeed, if the bubble wake contributions are neglected in Eq. 14 (i.e.,  $x = k = 0$ ), the following simplified relation is obtained:

$$\epsilon_L = \left[ \frac{U_L}{U_T(1 - \epsilon_G)} \right]^{1/N} (1 - \epsilon_G) \quad (20)$$

This relation can be seen as an extension to the three-phase fluidization of the relation of Richardson and Zaki (Eq. 3), obtained for liquid-solid fluidized beds. It suffices to replace in Eq. 3 the superficial liquid velocity by  $U_L/(1 - \epsilon_G)$  and the liquid holdup by  $\epsilon_L/(1 - \epsilon_G)$ . In the absence of a wake, a three-phase fluidized bed can be described as two separate flows: one liquid through particles and the other gas.

Using the experimental values for  $\epsilon_G$  and  $N$  calculated from relation 6, the predicted liquid holdup values from Eq. 20 are somewhat lower than the experimental data. However, the average relative deviations between the predictions and the data for the 2.0, 3.9, and 6.0 mm particle beds are equal to 11, 6.5 and 3.5%, respectively. This is surprisingly good and the fit improves as larger particle diameters are used. This is an indication that the simplified relation (Eq. 20) is more valid in the bubble disintegration regime, observed for the 6.0 mm particle bed.

If the bubble wake parameters of a three-phase fluidized bed are unknown, it is suggested to use Eq. 20 as a simplified relation between liquid and gas holdups.

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## Notation

- $D_C$  = column diameter, m
- $d_p$  = particle diameter, m
- $g$  = gravity, m/s<sup>2</sup>
- $k$  = ratio of the bubble wake volume to the bubble gas volume
- $N$  = exponent in the relation of Richardson and Zaki
- $Re_T$  = Reynolds number based on the terminal velocity, Eq. 5
- $T$  = temperature, °C
- $U_G$  = superficial gas velocity, m/s
- $U_{GL}$  = slip velocity, m/s
- $U_L$  = superficial liquid velocity, m/s
- $U_T$  = terminal velocity of a particle, m/s

## Greek letters

- $\epsilon$  = porosity =  $\epsilon_G + \epsilon_L$
- $\epsilon_G$  = gas holdup
- $\epsilon_L$  = liquid holdup
- $\epsilon_S$  = solids holdup

- $\epsilon_{SF}$  = solids holdup in the liquid-solid region
- $\epsilon_{SW}$  = solids holdup in the wake
- $\epsilon_W$  = volume fraction of the wake
- $\mu_L$  = viscosity of the liquid, Pa · s
- $\rho_G$  = gas density, kg/m<sup>3</sup>
- $\rho_L$  = liquid density, kg/m<sup>3</sup>
- $\rho_S$  = solids density, kg/m<sup>3</sup>

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